

NUMBER SEQUENCES

PATTERNS

TWO DIFFERENT CHAPTERS WITH
DIFFERENT TECHNIQUES.

ARITHMETIC SEQUENCE

$$4, \xrightarrow{+3} 7, \xrightarrow{+3} 10, \xrightarrow{+3} 13, \underline{16}, \underline{19} \quad d = 19 - 16 = +3$$

$$20, \xrightarrow{-2} 18, \xrightarrow{-2} 16, \xrightarrow{-2} 14, \underline{12}, \underline{10} \quad d = 14 - 16 = -2$$

Value of term
↑

$$n\text{th term} = a + (n-1)d$$

Common Difference
→ $d = \text{Next Term} - \text{Previous Term}$

n tells us the position number

$n\text{th term}$ tells us value written on that position

↓
First Term

↓
Term number

Q:

$$4, \xrightarrow{+3} 7, \xrightarrow{+3} 10, \xrightarrow{+3} 13, \underline{16}, \underline{19} \quad d = 16 - 13 = +3$$

- (i) Write down next two terms:
(ii) Find an expression, in terms of n , for $n\text{th term}$.

$$\begin{aligned} n\text{th term} &= a + (n-1)d \\ &= 4 + (n-1)(3) \\ &= 4 + 3n - 3 \end{aligned}$$

$$n\text{th term} = 3n + 1$$

- (iii) Find 50th term of this sequence.

$$n\text{th term} = 3n + 1$$

$$50^{\text{th}} \text{ term} = 3(50) + 1$$

$$50^{\text{th}} \text{ term} = 151$$

(iv) Given that n^{th} term of the sequence is 61, find n .

$$n^{\text{th}} \text{ term} = 3n + 1$$

$$61 = 3n + 1$$

$$60 = 3n$$

$$n = \frac{60}{3} = 20$$

$$\begin{array}{ccccccc} 4, & 7, & 10, & 13, & \dots & 61 \\ n=1 & n=2 & n=3 & n=4 & & n=? \end{array}$$

(v) Given that K^{th} term of the sequence is 91, find K .

$$n^{\text{th}} \text{ term} = 3n + 1$$

$$K^{\text{th}} \text{ term} = 3K + 1$$

$$91 = 3K + 1$$

$$90 = 3K$$

$$K = 30$$

$$\begin{array}{ccccccc} 4, & 7, & 10, & 13, & \dots & 91 \\ n=1 & n=2 & n=3 & n=4 & & n=K \end{array}$$

IF THE SEQUENCE IS NOT ARITHMETIC,
THEN IT WILL BELONG TO ONE OF THESE
THREE FAMILIES

SQUARE	CUBE	TRIANGLE	
1	1	1	○
4	8	3	○○○
9	27	6	○○○ ○○○
16	64	10	○○○ ○○○ ○○○
⋮	⋮	15	
nth term = n^2	n^3	$\frac{1}{2}n(n+1)$	

THESE nth terms only work if these sequences
start from 1.

IF THEY ARE NOT STARTING FROM 1:

SHIFTS

1. Forward shift: Number starts after skipping
first term(s)

So we replace $n \longrightarrow n+1$ etc

2. Backward shift: Number starts before
first term i.e 0.

So we replace $n \longrightarrow n-1$

Q. Write down n th term of this sequence.

4, 9, 16, 25

SQUARE: ORIGINAL: 1 4 9 16 25 n^2

SHIFT 4 9 16 25 ... $(n+1)^2$

$n \rightarrow n+1$

Ans. $(n+1)^2$

Q. Write down n th term of this sequence.

8, 27, 64, 125

Cube: Original: 1, 8, 27, 64, 125 n^3

SHIFT : 8, 27, 64, 125 $(n+1)^3$

FORWARD

$n \rightarrow n+1$

Ans. $(n+1)^3$

Q. Write down n th term of this sequence.

27, 64, 125

Cube: Original: 1, 8, 27, 64, 125 n^3

SHIFT : 27, 64, 125 $(n+2)^3$

FORWARD

$n \rightarrow n+2$

Ans. $(n+2)^3$

Q. Write down n th term of this sequence.

0, 1, 8, 27, 64, ...

Cube: Original: 1, 8, 27, 64, 125, ... n^3

Back shift: 0, 1, 8, 27, 64, ... $(n-1)^3$
 $n \rightarrow n-1$

Ans. $(n-1)^3$

Q. Write down n th term of this sequence.

0, 1, 4, 9, 16, 25, ...

SQUARE: 1, 4, 9, 16, ... n^2

BACK
SHIFT
 $n \rightarrow (n-1)$
0, 1, 4, 9, 16, ... $(n-1)^2$

Ans. $(n-1)^2$

Q. Write down n th term of this sequence.

3, 6, 10, 15, ...

TRIANGLE: 1, 3, 6, 10, 15, ... $\frac{1}{2}n(n+1)$

Forward Shift: 3, 6, 10, 15, ... $\frac{1}{2}(n+1)(n+1+1)$
 $n \rightarrow n+1$

$$\boxed{\frac{1}{2}(n+1)(n+2)}$$

Q write down n th term of this sequence.

0, 1, 3, 6, 10, 15, ...

TRIANGLE : 1 3 6 10 15 $\frac{1}{2}n(n+1)$

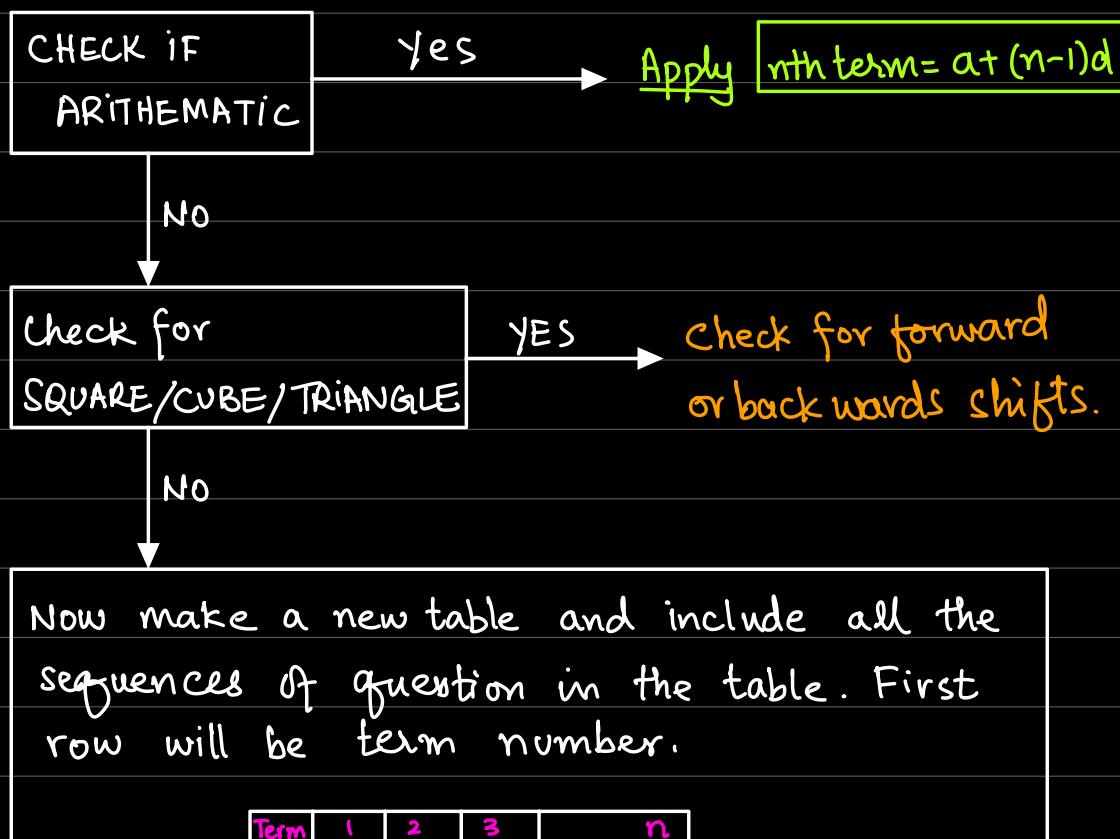
Back Shift
0 1 3 6 10 $\frac{1}{2}(n-1)(n-1+1)$

$n \rightarrow n-1$

$$\frac{1}{2}(n-1)(n) \quad \checkmark$$

$$\frac{1}{2}(n)(n-1) \quad \checkmark$$

EXAM TECHNIQUE



- 13 The first four terms of a sequence are 55, 53, 49, 41.
The n th term of this sequence is $57 - 2^n$.

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

- (a) Calculate the fifth term.

$$\begin{aligned} n\text{th term} &= 57 - 2^n \\ 5^{\text{th}} \text{ term} &= 57 - 2^5 \\ &= 57 - 32 \\ &= 25 \end{aligned}$$

Answer [1]

- (b) Write down the n th term of the sequence 56, 55, 52, 45

Term	1	2	3	4	
(a)	55	53	49	41	$57 - 2^n$
(b)	56	55	52	45	$n + 57 - 2^n$

Answer [1]

Number Sequences

↳ ARITHMETIC $\rightarrow$ nth term $= a + (n-1)d$

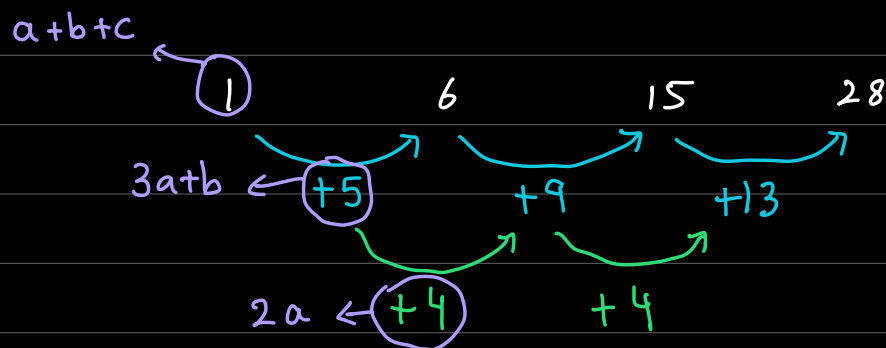
↳ SQUARE/CUBE/TRIANGLE $\rightarrow$ SHIFTS CHECK
 n^2 n^3 $\frac{1}{2}n(n+1)$

↳ VERTICAL PATTERN.

↳ QUADRATIC + EXPONENTIAL.

QUADRATIC SEQUENCES:
(SECOND DIFFERENCE IS SAME).

$$\text{nth term} = an^2 + bn + c$$



$$\begin{array}{l|l|l} 2a = 4 & 3a + b = 5 & a + b + c = 1 \\ a = 2 & 3(2) + b = 5 & 2 + (-1) + c = 1 \\ & 6 + b = 5 & 1 + c = 1 \\ & b = -1 & c = 0 \end{array}$$

$$n\text{th term} = an^2 + bn + c$$

$$= 2n^2 - 1n + 0$$

$$\boxed{n\text{th term} = 2n^2 - n}$$

Q: Find n th term of this:

$$\begin{array}{ccccccc}
 a+b+c = \textcircled{-2} & & -1 & & 4 & & 13 & & 26 \\
 & \nearrow & & \nearrow & & \nearrow & & \nearrow & \\
 3a+b = \textcircled{+1} & & +5 & & +9 & & +13 & & \\
 & \nearrow & & \nearrow & & \nearrow & & & \\
 2a = \textcircled{+4} & & +4 & & +4 & & & &
 \end{array}$$

$$\begin{array}{lcl}
 2a = 4 & , & 3a + b = 1 & , & a + b + c = -2 \\
 a = 2 & , & 3(2) + b = 1 & , & 2 + (-5) + c = -2 \\
 & & b = -5 & & -3 + c = -2 \\
 & & & & c = 1
 \end{array}$$

$$n\text{th term} = an^2 + bn + c$$

$$n\text{th term} = 2n^2 - 5n + 1$$

EXPONENTIAL (Power)

$$1) \quad 2 \quad 4 \quad 8 \quad 16 \quad 32 \dots$$

$$2^1 \quad 2^2 \quad 2^3 \quad 2^4 \quad 2^5 \dots 2^{\boxed{n}}$$

Powers: 1, 2, 3, 4, ..., n

$$2) \quad \begin{array}{cccccc} 1 & 2 & 4 & 8 & 16 & \\ 2^0 & 2^1 & 2^2 & 2^3 & 2^4 & \longrightarrow 2^{n-1} \end{array}$$

power: $0, 1, 2, 3 \dots$ (Arithmetic)

$$\begin{aligned} \text{nth term} &= a + (n-1)d \\ &= 0 + (n-1)(1) \\ &= n-1 \end{aligned}$$

$$3) \quad \begin{array}{ccccccc} \frac{1}{9} & \frac{1}{3} & 1 & 3 & 9 & 27 & \\ 3^{-2} & 3^{-1} & 3^0 & 3^1 & 3^2 & 3^3 & \longrightarrow 3^{n-3} \end{array}$$

power: $-2, -1, 0, 1, 2, 3 \dots$ (Arith)

$$\begin{aligned} &= a + (n-1)d \\ &= -2 + (n-1)(+1) \\ &= -2 + n - 1 \\ &= n - 3 \end{aligned}$$

4)

1, 8, 64, ...

$2^0, 2^3, 2^6, \dots$

2^{3n-3}

Powers: 0, 3, 6, ... Arithmetic

$$a + (n-1)d$$

$$0 + (n-1)(3)$$

$$3n-3$$

PATTERNS: These are things that usually stay same.

23 A pattern of numbers is given below.

Row 1 $\rightarrow \frac{1}{1 \times 2} = \frac{1}{1} - \frac{1}{2}$

$$\frac{1}{\square \times \square} = \frac{1}{\square} - \frac{1}{\square}$$

Row 2 $\rightarrow \frac{1}{2 \times 3} = \frac{1}{2} - \frac{1}{3}$

Row 3 $\rightarrow \frac{1}{3 \times 4} = \frac{1}{3} - \frac{1}{4}$

Row 4 $\rightarrow \frac{1}{4 \times 5} = \frac{1}{4} - \frac{1}{5}$

(a) Write down Row 10.

Answer $\frac{1}{10 \times 11} = \frac{1}{10} - \frac{1}{11}$ [1]

(b) Adding the first two rows gives the result $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} = \frac{1}{1} - \frac{1}{3} = \frac{2}{3} \rightarrow \text{Rows} \rightarrow \text{Rows}+1$
 Adding the first three rows gives the result $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} = \frac{1}{1} - \frac{1}{4} = \frac{3}{4} \rightarrow \text{Rows} \rightarrow \text{Rows}+1$

(i) Write down the result of adding the first four rows.

Answer $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} = \frac{1}{1} - \frac{1}{5} = \frac{4}{5} \rightarrow \text{Rows} \rightarrow \text{Rows}+1$ [1]

(ii) Use the pattern to write down

(a) the value of $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{19 \times 20} = \frac{19}{20} \rightarrow \text{Rows} \rightarrow \text{Rows}+1$

Answer $\frac{19}{20}$ [1]

(b) the number of rows that add up to $\frac{109}{110} \rightarrow \text{Rows} \rightarrow \text{Rows}+1$

Answer 109 [1]

(c) an expression, in terms of n , for the result of adding the first n rows.

$$\frac{\text{Rows}}{\text{Rows}+1} = \frac{n}{n+1}$$

Answer [1]

10 Look at this pattern. $\rightarrow \square^2 - \square^2 = 4 \times \square$

Line 1 $2^2 - 0^2 = 4 \times 1$

Line 2 $3^2 - 1^2 = 4 \times 2$

Line 3 $4^2 - 2^2 = 4 \times 3$

Line 4 $5^2 - 3^2 = 4 \times 4$

First:

2, 3, 4, 5, ... Arithmetic

$$a + (n-1)d$$

$$2 + (n-1)(1)$$

$$2 + n - 1 = \boxed{n+1}$$

Second: 0, 1, 2, 3, ... Arith

$$a + (n-1)d$$

$$0 + (n-1)(1)$$

$$(n-1)$$

Last: 1, 2, 3, ... $\boxed{n}$

(a) Write down the 7th line of the pattern.

Answer (a) $\boxed{8}^2 - \boxed{6}^2 = 4 \times \boxed{7}$ [1]

(b) Write down the n th line of the pattern.

Answer (b) $\boxed{n+1}^2 - \boxed{n-1}^2 = 4 \times \boxed{n}$ [1]

(c) Use the pattern to find $521^2 - 519^2$.

$$\begin{aligned} \boxed{521}^2 - \boxed{519}^2 &= ?? \\ \boxed{n+1}^2 - \boxed{n-1}^2 &= 4 \times \boxed{n} \\ \downarrow & \quad \downarrow \\ n+1 &= 521 \\ n &= 520 \end{aligned} \quad \begin{aligned} ?? &= 4 \times n \\ &= 4 \times 520 \\ &= 2080 \end{aligned}$$

Answer (c) [1]

(d) Use the pattern to find the positive integers x and y such that $x^2 - y^2 = 484$.

$$\begin{aligned} \boxed{x}^2 - \boxed{y}^2 &= 4 \times \boxed{n} \\ \boxed{x}^2 - \boxed{y}^2 &= 484 \\ x &= n+1, \quad y = n-1, \quad 4 \times n = 484 \\ n &= \frac{484}{4} \\ n &= 121 \\ x &= 121+1 \\ y &= 121-1 \\ \boxed{x} &= 122 \\ \boxed{y} &= 120 \end{aligned}$$

Answer (d) $x = \boxed{122}$
 $y = \boxed{120}$ [1]

- 19 The first four lines of a pattern of numbers are shown below.

$$\text{1st line} \quad 3^2 - 1^2 = 8 \times 1$$

$$\text{2nd line} \quad 5^2 - 1^2 = 8 \times (1 + 2)$$

$$\text{3rd line} \quad 7^2 - 1^2 = 8 \times (1 + 2 + 3)$$

$$\text{4th line} \quad 9^2 - 1^2 = 8 \times (1 + 2 + 3 + 4)$$

- (a) Write down the 7th line of the pattern.

Answer [1]

- (b) Write down an expression, in terms of n , to complete the n th line of the pattern.

Answer $= 8 \times (1 + 2 + 3 + 4 + \dots + n)$ [1]

- (c) Using the n th line of the pattern, show that $1 + 2 + 3 + 4 + \dots + n = \frac{n(n+1)}{2}$.